

Type Assisted Synthesis of Programs with Algebraic Data Types

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Example - Desugaring a simple language

ADT Definitions

```
adt srcAST {  
  NumS { int v; }  
  TrueS { }  
  FalseS { }  
  BinaryS { opcode op; srcAST a; srcAST b; }  
  BetweenS { srcAST a; srcAST b; srcAST c; }  
}
```

```
adt dstAST {  
  NumD { int v; }  
  BoolD { bit v; }  
  BinaryD { opcode op; dstAST a; dstAST b; }  
}
```



a < b < c

Example - Desugaring a simple language

ADT Definitions

```
adt srcAST {  
  NumS { int v; }  
  TrueS { }  
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  BetweenS { srcAST a; srcAST b; srcAST c; }  
}  
  
adt dstAST {  
  NumD { int v; }  
  BoolD { bit v; }  
  BinaryD { opcode op; dstAST a; dstAST b; }  
}
```

Specification

$\text{interpretSrc}(s) == \text{interpretDst}(\text{desugar}(s))$

Example - Desugaring a simple language

Function to be synthesized

```
dstAST desugar(srcAST s) {
```

```
}}
```

Example - Desugaring a simple language

Function to be synthesized

```
dstAST desugar(srcAST s) {  
  if (s == null) return null;  
  switch(s){  
    repeat_case: {  
      dstAST a = desugar(s.??);  
      dstAST b = desugar(s.??);  
      dstAST c = desugar(s.??);  
      return ??(3, {a, b, c, s.??});  
    }  
  }  
}
```

repeat_case
General structure of pattern matching

Example - Desugaring a simple language

Function to be synthesized

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  }  
}}
```

Field selector hole
Choice among fields of s

Example - Desugaring a simple language

Function to be synthesized

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  }  
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```

Generalized constructor
Construct arbitrary ADT tree of depth at most 3

Example - Desugaring a simple language

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      return ??(3, {a, b, c, s.??});  
    }  
  }  
}
```

LOC: 7

No. of possible functions from template ~ 2^{110}

Example - Desugaring a simple language

Output

```
dstAST desugar(srcAST s) {
  if (s == null) return null;
  switch(s){
  case NumS: { return new NumD(v = s.v); }
  .....
  case BetweenS: {
    dstAST a = desugar(s.a);
    dstAST b = desugar(s.b);
    dstAST c = desugar(s.c);
    return new BinaryD(op = new AndOp(),
      a = new BinaryD(op = new LtOp(),
        a = a, b = b),
      b = new BinaryD(op = new LtOp(),
        a = b, b = c)); }
  .....
  } LOC: 22
```

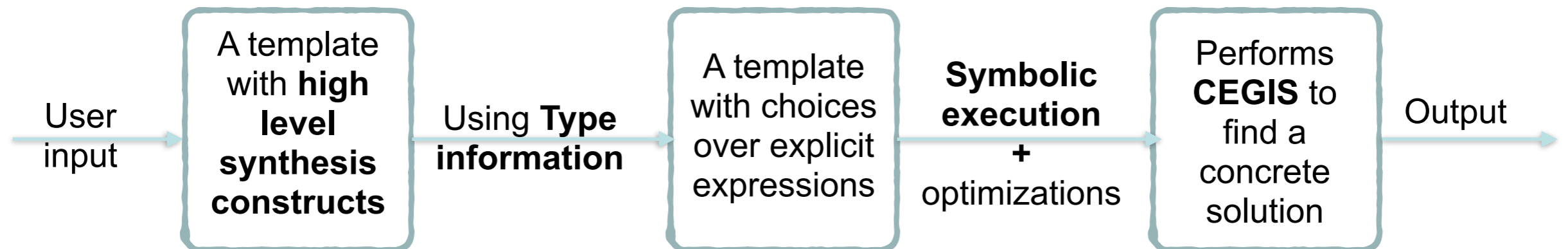
Synthesis Time: 36s

Technical Challenges

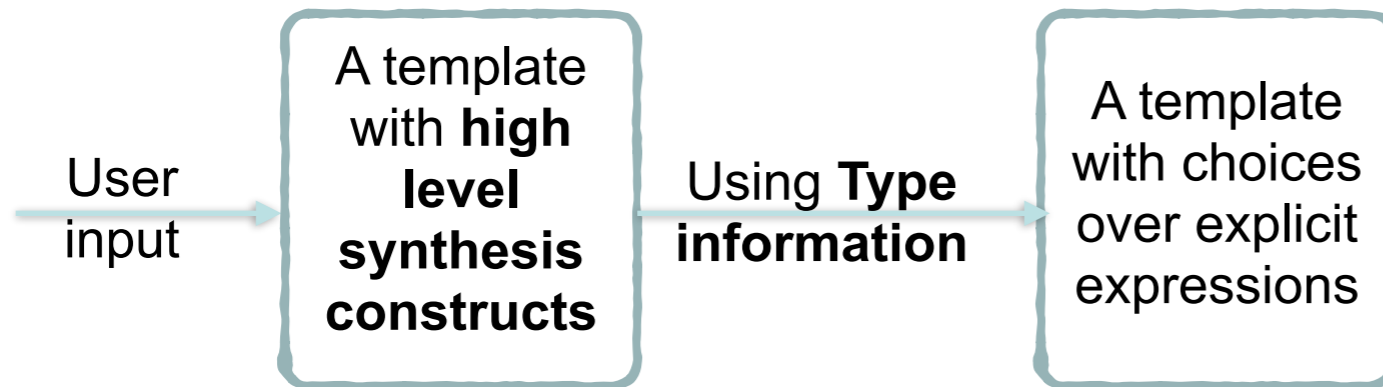
- Huge search space
 - On the order of $2^{100} - 2^{500}$
- Complex specifications
 - Like interpreters
- High degree of recursion

Approach

- Constraint based synthesis using Sketch [1]



Type Directed Transformation

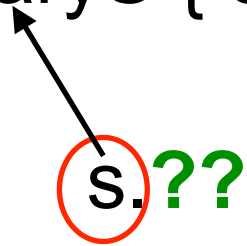


Type Directed Transformation

Requires propagating type information both top-down and bottom-up

```
BinaryS { opcode op; srcAST a; srcAST b; }
```

S??



Type Directed Transformation

Requires propagating type information both top-down and bottom-up

```
BinaryS { opcode op; srcAST a; srcAST b; }
```



Type Directed Transformation

- Bi-directional rules of Pierce and Turner, 2000

$$\frac{T = \{\tau_0 \dots \tau_i\}}{\Gamma \vdash e \xrightarrow{T} \{e_0 \dots e_k\}}$$

General transformation rule

Type Directed Transformation

$$\frac{\begin{array}{l} T' = \{ \tau \mid \tau \text{ has a field } l : \tau_i \text{ and } \tau_i \in T \} \\ \Gamma \vdash e \xrightarrow{T'} \{e_0 \dots e_k\} \end{array}}{\Gamma \vdash e.?? \xrightarrow{T} \{e_i.l_j \mid e_i.l_j : \tau \ j \in [0, k] \text{ and } \tau \in T\}}$$

Type Directed Transformation

$$\frac{\begin{array}{l} T' = \{ \tau \mid \tau \text{ has a field } l : \tau_i \text{ and } \tau_i \in T \} \\ \Gamma \vdash e \xrightarrow{T'} \{e_0 \dots e_k\} \end{array}}{\Gamma \vdash e.?? \xrightarrow{T} \{e_i.l_j \mid e_i.l_j : \tau \ j \in [0, k] \text{ and } \tau \in T\}}$$

$$s.?? \xrightarrow{\{\text{srcAST}\}} \longrightarrow$$

Type Directed Transformation

$$\frac{\begin{array}{c} T' = \{ \tau \mid \tau \text{ has a field } l : \tau_i \text{ and } \tau_i \in T \} \\ \Gamma \vdash e \xrightarrow{T'} \{e_0 \dots e_k\} \end{array}}{\Gamma \vdash e.?? \xrightarrow{T} \{e_i.l_j \mid e_i.l_j : \tau \ j \in [0, k] \text{ and } \tau \in T\}}$$

$$T' = \{\text{BinaryS}, \text{BetweenS}\}$$

$$s.?? \xrightarrow{\{\text{srcAST}\}}$$

Type Directed Transformation

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$$T' = \{\text{BinaryS}, \text{BetweenS}\}$$

$$s \xrightarrow{\{\text{BinaryS}, \text{BetweenS}\}} \{s\}$$

$$s.?? \xrightarrow{\{\text{srcAST}\}}$$

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$$s \xrightarrow{\{\text{BinaryS}, \text{BetweenS}\}} \{s\}$$

$$s.?? \xrightarrow{\{\text{srcAST}\}} \{s.a, s.b\}$$

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$$T' = \{ \tau \mid \tau \text{ has a field } l : \tau_i \text{ and } \tau_i \in T \}$$
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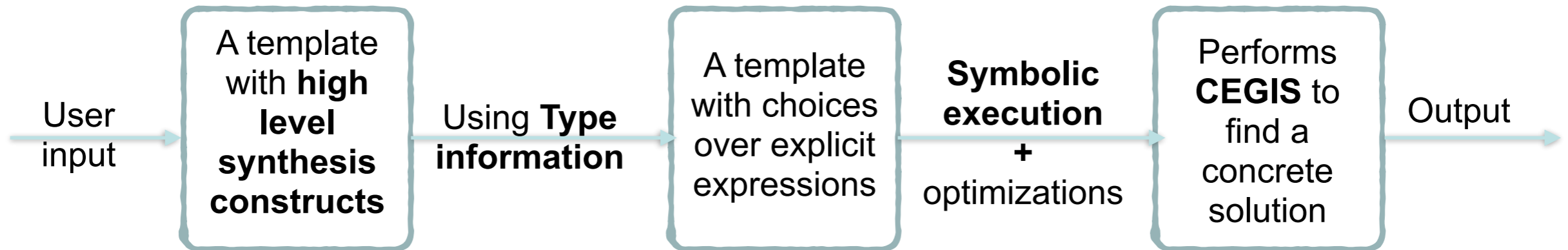
$$T' = \{\text{BinaryS}, \text{BetweenS}\}$$

$$s \xrightarrow{\{\text{BinaryS}, \text{BetweenS}\}} \{s\}$$

$$s.?? \xrightarrow{\{\text{srcAST}\}} \{s.a, s.b\}$$

$$s.???.??$$

Synthesis



Synthesis

- Inlines function calls and unrolls loops and creates a formula to encode to the SAT solver
- Uses Counter Example Guided Inductive Synthesis (CEGIS)
- Optimizations to improve scalability

Optimizations

1. Merging recursive calls with mutually exclusive path conditions

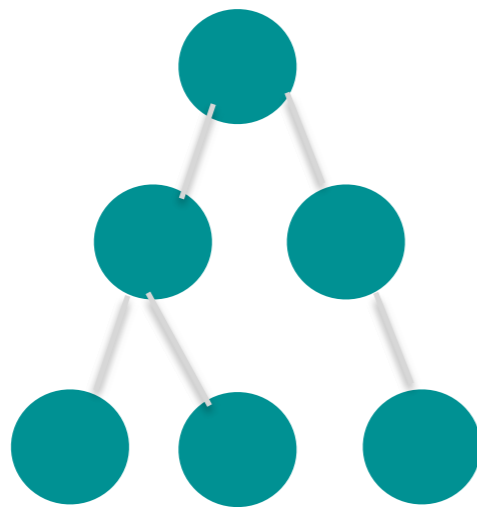
```
if (w)  
    x = foo(a, b);  
else  
    y = foo(c, d);
```



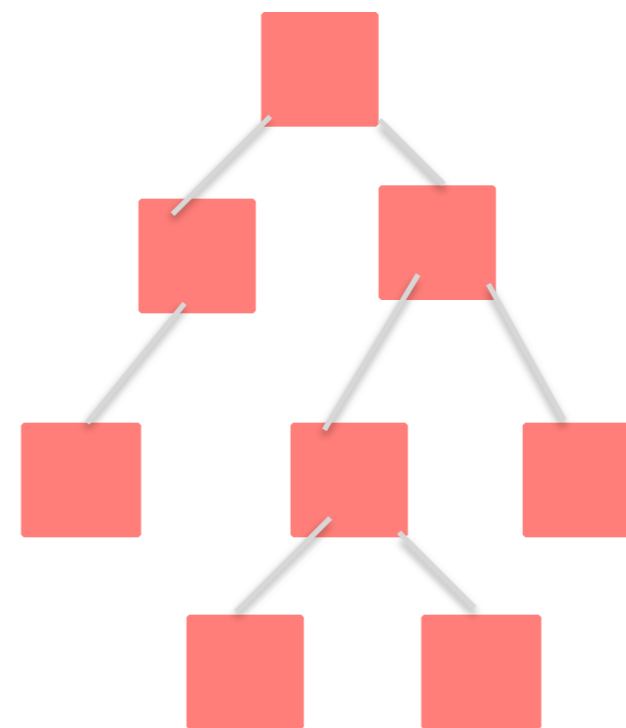
```
t = foo(w?a:c, w?b:d);  
if (w)  
    x = t;  
else  
    y = t;
```


Optimizations

2. Use specification as an invariant to abstract recursive calls



Input AST node

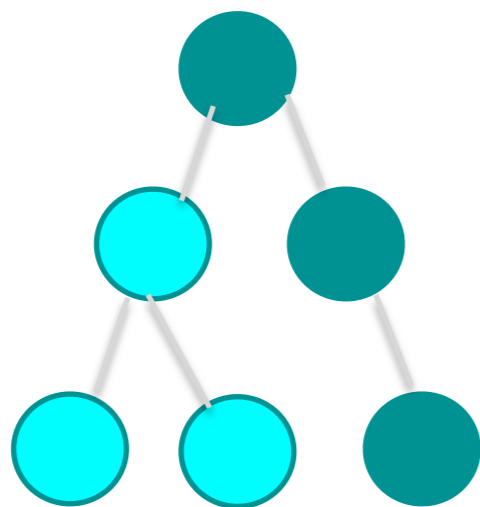


Desugared AST node

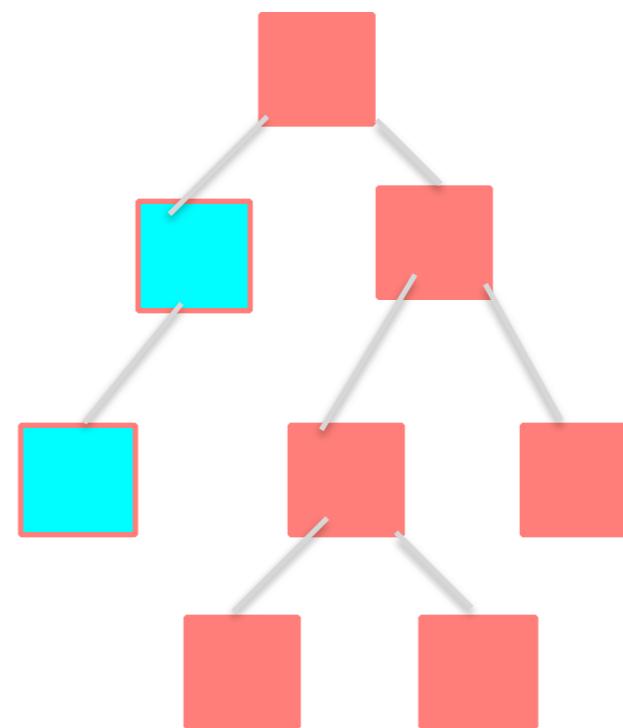
Specification: $\text{interpretSrc}(s) == \text{interpretDst}(\text{desugar}(s))$

Optimizations

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Input AST node

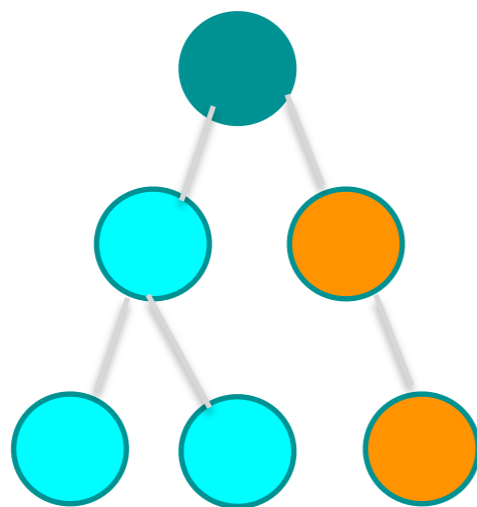


Desugared AST node

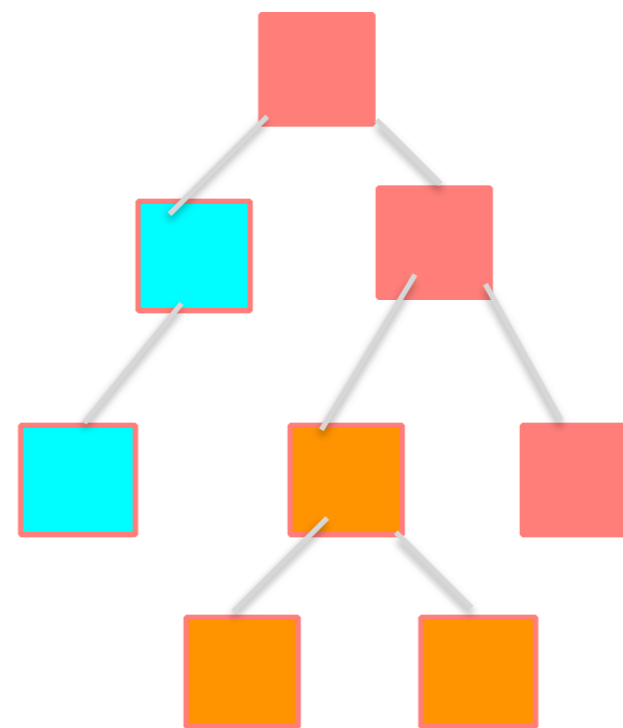
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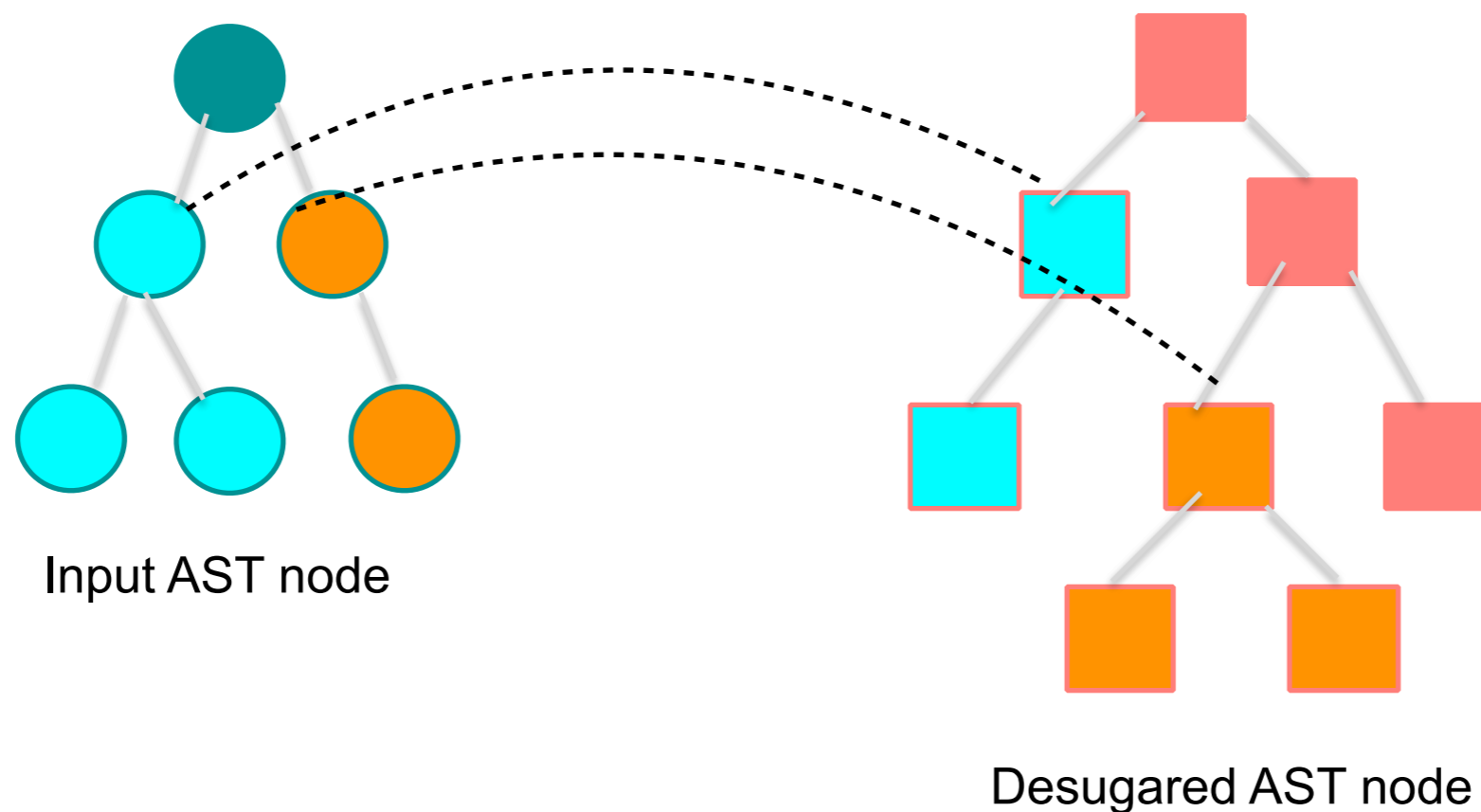


Desugared AST node

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Optimizations

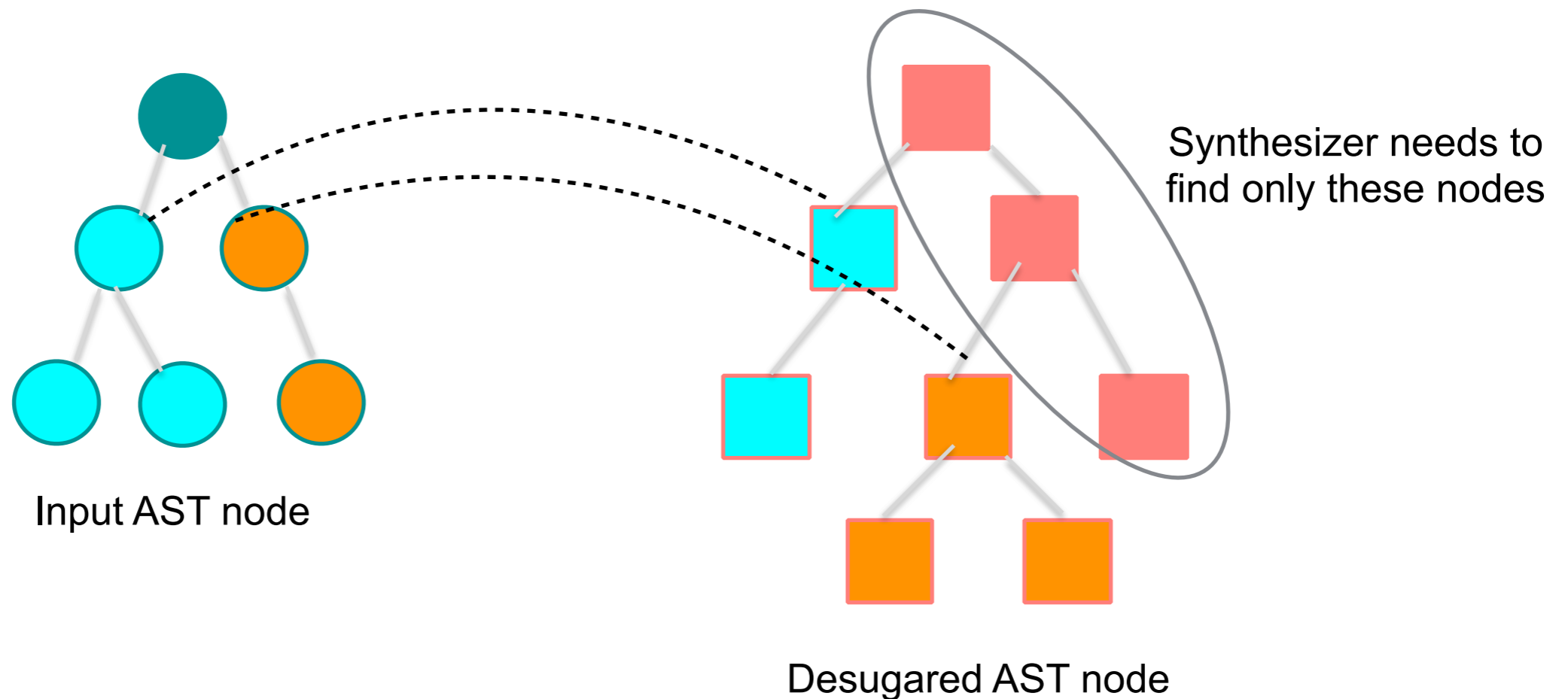
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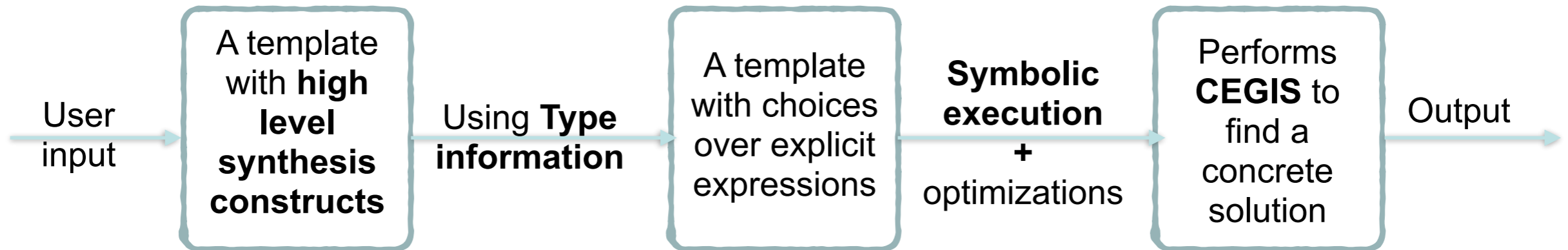
Optimizations

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Synthesis



Evaluation

Benchmark	Runtime (s)	# of program choices
Insertion into a binary tree	18.42	2^{72}
Simple language desugaring	36.36	2^{110}
Simple language desugaring with state	577.19	2^{141}
Booleans to Lambda Calculus	114.14	2^{541}
Pairs to Lambda Calculus	683.55	2^{183}
AST optimizations	163.09	2^{162}
Type constraints for Lambda Calculus	496.12	2^{149}

Conclusion

- A system to synthesize functions on Algebraic Data Types from high level templates
- Uses a combination of type inference and symbolic solving
- Can synthesize complex functions like desugaring

THANK YOU